(i) Printed Pages: 2 Roll No. .....

(ii) Questions : 8 Sub. Code : 0 4 4 4

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

1128

MATHEMATICS

Paper—II : Modern Algebra

Time Allowed : Three Hours] [Maximum Marks : 30

Note :— Attempt *five* questions in all, selecting at least *two* questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Define a group. Give an example of a finite non-abelian group.

   (b) Define order of an element of a group and prove that in a finite group the order of every element exists. 3,3

2. (a) Let G be a group and a, b ∈ G. Is O(ab) = O(a). O(b), in general. Justify.

   (b) If H and K be any two subgroups of group G, then prove that HK is a subgroup of G iff HK = KH. 2,4

3. (a) Define normalizer of an element of a group. Prove that it is a subgroup of G.

   (b) Prove that subgroup of a cyclic group is cyclic. 3,3

4. (a) Find cosets of $2\mathbf{Z}$ in $\mathbf{Z}$, the set of integers.

(b) Prove that infinite cyclic group is isomorphic to the additive group of integers.

(c) Evaluate f^{-1} gf where f = (1 3 5) (1 2) and g = (1 5 7 9). 2,2,2

SECTION—B

5. Give an example of the following with proper reasoning :

(a) A commutative ring without unity

(b) A ring with zero divisors

(c) A division ring. 2,2,2

6. (a) Prove that a division ring has no proper ideal.

(b) Let E be the ring of even integers. Prove that <4> is a maximal ideal in E. 3,3

7. (a) Let R be a commutative ring with unity and M be an ideal of R. Prove that M is maximal ideal of R if and only if R/M is a field.

(b) Show that the polynomial $x^2 + x + 4$ is irreducible over Z_{11}, the field of integers modulo 11. 3,3

8. (a) Let f : R → S be a homomorphism function from R onto S and K be the kernel of f. Prove that :

$$R/K \cong S.$$

(b) What are the units of the polynomial ring $Z_7[x]$? 3,3

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